O SCALE MODEL RAILROADING
ADVANCED STEM CURRICULUM

Grades 11–12 & Freshman College

A Rigorous, Project-Based Exploration of Electrical Engineering,
Applied Physics, Computational Geometry & Structural Analysis

O Scale Advanced STEM Curriculum

Series Title	Precision Engineering: Advanced STEM Through O Scale Model Railroading
Target Audience	Grades 11–12 (AP Physics, Pre-Calculus, Engineering Design) | College Freshman (Intro EE, Statics, Engineering Graphics)
Module Count	4 Modules + Capstone Research Project | 90–120 min per module
Prerequisites	Algebra II; introductory physics (forces, energy); basic familiarity with DC circuits recommended but not required
NGSS / Standards	HS-PS2, HS-PS3, HS-ETS1; ABET Student Outcomes (a)(b)(c)(e)(g)(k); CCSS-Math HSN, HSA, HSF
Module 1	Advanced Electrical Engineering — DCC, PWM, Signal Integrity & Layout Power Systems
Module 2	Applied Physics & Dynamics — Tractive Effort, Davis Equation, Braking & Energy Recovery
Module 3	Computational Geometry & CAD — 3D Track Planning, Superelevation & Clearance Analysis
Module 4	Structural Engineering — Load Analysis, Finite Element Concepts & Bridge Optimization
Capstone	System Integration Design Project — Full Layout Engineering Package
Scale	O Scale (1:48) — all calculations include prototype scaling to real-world dimensions
Contact	O Scale STEM Education Initiative

> **Curriculum Philosophy:** This curriculum treats the model railroad not as a toy but as a scaled engineering system. Every calculation performed on the model is immediately scaled to prototype dimensions and compared against published real-world specifications. Students learn that engineering is not abstract — it is the precise, quantitative management of physical reality. The O Scale layout is the laboratory.

Scope & Sequence

Each module follows a consistent five-phase structure: (1) Industry Context — situating the concept in real railroad engineering practice; (2) Theory — rigorous mathematical and scientific foundations; (3) Experimental Investigation — hypothesis-driven lab work on the actual layout; (4) Analysis & Modeling — computational analysis, graphing, and comparison to theoretical predictions; and (5) Professional Communication — written reports and presentations in engineering document format.

The capstone project requires students to produce a complete Engineering Design Package for a specified layout scenario — integrating all four module competencies into a single, coherent, professionally formatted deliverable.

Suggested Teaching Contexts

- AP Physics C (Electricity & Magnetism; Mechanics) — Modules 1 & 2 as laboratory supplements
- Pre-Calculus / Calculus — Module 2 (rates of change, optimization) and Module 3 (parametric curves, coordinate geometry)
- Engineering Design / PLTW — all four modules as a semester elective arc
- College Freshman: Introduction to Electrical Engineering — Module 1 as a hands-on circuit lab
- College Freshman: Engineering Statics / Mechanics of Materials — Module 4 as a bridge design lab
- Dual Enrollment — complete four-module series as a 1-credit Engineering Applications course

MODULE 1

Advanced Electrical Engineering

DCC Architecture, PWM Motor Control, Signal Integrity & Layout Power Systems

Duration	120 minutes (or two 60-min sessions)
Level	Grade 11–12 (AP Physics C: E&M) | College Freshman (Intro EE)
Core Standards	HS-PS3-3: Design, build, and refine a device that works within given constraints to convert one form of energy to another | HS-ETS1-2: Design solutions under constraints
ABET Outcomes	(a) Apply math, science, engineering knowledge; (b) Design and conduct experiments; (k) Use modern engineering techniques and tools
Key Concepts	PWM (Pulse Width Modulation), DCC packet structure, impedance matching, voltage drop over distance, Kirchhoff's Laws, power budgeting, short-circuit protection
Required Math	Ohm's Law, Power equations ($P=IV$, $P=I^2R$), Kirchhoff's Current & Voltage Laws, basic binary/hex notation for DCC addresses
Driving Question	How does a single pair of rails simultaneously deliver power to multiple locomotives and carry digital command data — without confusion or interference?

Industry Context

The Digital Command Control (DCC) standard (S-9.1 through S-9.2.3) was developed beginning in the early 1990s and represents a genuine electrical engineering achievement: it superimposes a digital data stream on an AC power carrier, allowing dozens of independently addressed decoders to coexist on a single two-wire bus. The protocol is now an international standard (IEC 62756-1). Understanding DCC requires mastery of PWM, digital signal encoding, impedance, and power distribution — the same competencies demanded in automotive electronics, industrial motor control, and IoT device firmware.

Theoretical Foundations

1.1 — Pulse Width Modulation (PWM) & Motor Speed Control

Traditional DC model railroad control varies voltage to control speed. DCC replaces this with a fixed-amplitude AC waveform on which speed commands are encoded digitally. Inside the decoder, a microprocessor reconstructs motor voltage using PWM: a high-frequency square wave whose duty cycle determines average motor voltage.

```
V_avg = V_supply × (t_on / T)
```

Where t_on is the ON time per cycle and T is the total period. A 50% duty cycle produces half supply voltage; 25% produces quarter voltage. PWM frequency in DCC decoders typically ranges from 18 kHz to 40 kHz — above human hearing, reducing motor whine.

Derivation Exercise: If a DCC system operates at 14.5V RMS and a decoder runs PWM at 22 kHz with a 65% duty cycle, calculate: (a) V_avg delivered to the motor; (b) the period T in microseconds; (c) t_on and t_off. Then explain why running at higher PWM frequencies reduces audible motor noise while increasing switching losses in the H-bridge driver.

1.2 — DCC Packet Structure & Binary Encoding

DCC data is encoded as a Manchester-like biphase signal. A logical "1" is represented by a half-period of 58 μs; a logical "0" by a half-period of 100 μs. Each packet contains a preamble, address byte(s), data byte(s), and an error-detection byte (XOR checksum).

```
Packet = [Preamble: 14+ "1" bits] [0] [Address byte] [0] [Data byte] [0]
[Error byte] [1]
```

Students decode a sample DCC waveform trace and verify the checksum:

Decoding Exercise: Given a DCC packet with address byte 00000011 (address 3), instruction byte 01100101 (set speed step 37 forward), calculate the required error detection byte using XOR of address and instruction bytes. Verify: 00000011 XOR 01100101 = 01100110. Convert 01100110 to decimal and hex.

1.3 — Kirchhoff's Laws Applied to Layout Power Distribution

A large O Scale layout is not a simple circuit — it is a network. Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) govern how power distributes across multiple boosters, feeders, and sub-districts.

```
KVL: Σ V = 0 around any closed loop
KCL: Σ I_in = Σ I_out at any node
```

Voltage drop across feeder wires is a critical real-world problem. For a layout with 10 meters of 20-gauge wire (resistance ≈ 0.034 Ω/m), carrying 3A to a remote section:

```
V_drop = I × R_wire = 3A × (10m × 2 × 0.034 Ω/m) = 3 × 0.68 = 2.04V
```

This 2.04V drop on a 14.5V system represents a 14% loss — enough to cause erratic decoder behavior. Students calculate minimum wire gauge requirements for specified voltage drop budgets.

1.4 — Short Circuit Protection & Impedance

DCC boosters protect against short circuits using electronic circuit breakers. The response time must be fast enough to prevent decoder damage but slow enough to allow momentary shorts (e.g., a wheel bridging a rail gap) to self-clear.

The booster output impedance Z_out must be low relative to the decoder input impedance Z_in for maximum power transfer — a direct application of impedance matching theory from transmission line engineering.

```
Power Transfer Efficiency = Z_load / (Z_source + Z_load)
```

Laboratory Investigation

Experiment 1A — Feeder Wire Voltage Drop Mapping

Students place a precision voltmeter at intervals along a 20-foot O Scale layout powered by a single booster. Measure rail-to-rail voltage at 0 ft, 5 ft, 10 ft, 15 ft, and 20 ft from the booster connection. Load the circuit with a fixed resistor simulating two locomotives.

- Record voltage at each point.
- Calculate measured resistance per foot.
- Compare to theoretical resistance from wire gauge tables.
- Identify the "voltage sag" point and determine minimum number of feeder drops required to maintain $V \geq 12.5V$ everywhere.

Data table: Distance (ft) | Measured V | Theoretical V | Difference | % Error

Experiment 1B — PWM Oscilloscope Analysis

Using a digital oscilloscope (or PC-based oscilloscope software + sound card), capture the DCC waveform on the rails. Students measure:

1. Peak-to-peak voltage and RMS voltage.
2. Period T and frequency f of the carrier.
3. Width of "1" half-bits vs. "0" half-bits in microseconds.
4. Identify a complete packet in the waveform and decode the address byte.
5. Calculate the data rate in bits per second and compare to the DCC S-9.1 specification.

Advanced Extension: Model the DCC rail as a transmission line with distributed inductance L and capacitance C per unit length. Calculate the characteristic impedance $Z_0 = \sqrt{(L/C)}$. At what layout length does transmission line effects become significant? (Hint: compare layout length to signal wavelength $\lambda = v/f$ where $v \approx 2\times10^8$ m/s for typical wire.)

Experiment 1C — Power Budget Analysis

Students design a power budget for a hypothetical large O Scale layout with the following loads:

Load Element	Unit Draw	Quantity	Total Current (A)
Large steam locomotive (sound + lights + motor)	1.2A	3	
Diesel locomotive (DCC sound)	0.8A	4	
Passenger car (interior lighting, 6 LEDs)	0.05A	12	
Switch machine (momentary, average)	0.3A	8	

Accessories (signals, crossing gates, etc.)	0.1A	15	
Track bus resistance loss (estimated)	0.5A	—	

Students calculate total current demand, select appropriate booster capacity (with 25% safety margin), determine number of power districts needed (max 5A per district recommended), and specify wire gauges for each run.

Standards & Outcomes Alignment — Module 1

Standard / Code	Disciplinary Core Idea / Practice	How the O Scale Activity Addresses It
HS-PS3-3	Design device to convert energy within constraints	Students design a multi-district power system within specified voltage drop and current constraints
HS-PS3-5	Develop and use a model of two objects interacting through electric/magnetic fields	Students model the DCC signal as an electromagnetic wave propagating along the rail transmission line
HS-ETS1-2	Design solutions using systematic process to determine how well they meet criteria	Students evaluate 3 feeder configurations against cost, voltage drop, and installation complexity criteria
AP Physics C E&M	Circuits: Kirchhoff's Laws, RC circuits, magnetic induction	All three laboratory investigations directly exercise AP Physics C circuit analysis skills
ABET (a)	Apply knowledge of mathematics, science, and engineering	Students apply Ohm's Law, KVL/KCL, power equations, and binary arithmetic in a real engineering context
ABET (b)	Design and conduct experiments; analyze and interpret data	Experiments 1A, 1B, 1C all require experimental design, data collection, and comparison to theory

Materials — Module 1

Item	Qty / Group	Specification / Notes
O Scale DCC command station + booster (e.g., Digitrax Zephyr or NCE Power Cab)	1	DCC standard compliant
DCC-equipped O Scale locomotives (minimum 3, different decoders)	3	Mix of manufacturers to show decoder variation
Digital multimeter with true RMS capability	1 per group	Fluke 115 or equivalent
Digital oscilloscope OR PC oscilloscope (Analog Discovery 2 or Picoscope)	1 per group	2-channel minimum; 1 MHz+ bandwidth
Precision decade resistance box (0–100Ω, 0.1Ω steps)	1	For load simulation

22 AWG, 20 AWG, 18 AWG hookup wire samples, 10 ft each	1 set per group	For wire resistance comparison
Terminal blocks and bus wire (14 AWG, red/black)	1 set	For power district construction
O Scale layout with at least 20 ft of track in a single run	1	Pre-wired baseline; students add districts
Laptop with NCE or JMRI decoder programming software	1 per group	Free download; JMRI recommended
Lab notebook or engineering logbook (bound, quad-ruled)	1 per student	Professional engineering documentation practice

Assessment — Module 1

Criterion	4 – Exemplary	3 – Proficient	2 – Developing	1 – Beginning
KVL/KCL Analysis	Correctly applies both laws to a 3-node layout network; identifies all loops; calculates currents and voltages to <5% error	Correctly applies both laws to a 2-node network with minor arithmetic errors	Applies one law correctly; confuses voltage and current in the other	Cannot apply KVL or KCL without step-by-step guidance
PWM Calculation	Correctly calculates V_avg, duty cycle, t_on/t_off for 3+ scenarios; explains relationship to motor acoustics	Correct for 2 scenarios; partial explanation of acoustics	Sets up PWM formula correctly; arithmetic errors; no acoustic explanation	Cannot apply PWM formula without direct instruction
DCC Packet Decoding	Decodes complete packet including address, instruction, and checksum; identifies bit timing from oscilloscope trace	Decodes address and instruction bytes; minor checksum error	Decodes address byte only; cannot construct checksum	Cannot identify bit boundaries in the DCC waveform
Power Budget Design	Complete budget with all loads; correct safety margin; feeder gauge selected using voltage drop formula; districts justified	Complete budget; safety margin applied; feeder gauge selected without derivation	Budget complete but safety margin missing; wire gauge selected by guessing	Budget significantly incomplete; no wire gauge analysis

Lab Report Quality	Report in engineering format: abstract, theory, procedure, data, analysis, conclusion, references; all graphs properly labeled; error analysis included	Report complete; one or two format elements missing; graphs present but incomplete labels	Report present but informal; data present without analysis; no error analysis	Report is notes-only; no analysis or professional formatting

MODULE 2

Applied Physics & Dynamics

Tractive Effort, the Davis Equation, Braking Physics & Regenerative Energy

Duration	120 minutes
Level	Grade 11–12 (AP Physics C: Mechanics) | College Freshman (Engineering Dynamics)
Core Standards	HS-PS2-1: Analyze data to support Newton's Second Law | HS-PS3-1: Create a computational model of energy stored in fields | HS-ETS1-4: Use a computer simulation to model effects of proposed design solutions
ABET Outcomes	(a) Apply math/science/engineering; (b) Design experiments; (e) Identify, formulate, solve engineering problems
Key Concepts	Tractive effort curves, Davis Train Resistance Equation, adhesion coefficient, braking force, kinetic energy, regenerative braking, grade resistance, curve resistance
Required Math	Calculus (derivatives for acceleration, integrals for work/energy), unit conversion, dimensional analysis, graphing and curve fitting
Driving Question	Given a specified locomotive, a defined grade, and a target train tonnage — will the train make it over the hill? Prove it mathematically, then verify experimentally.

Industry Context

Train performance prediction is a core railroad engineering discipline. Before any locomotive is assigned to a route, engineers consult tractive effort curves — graphical representations of the locomotive's ability to move tonnage at various speeds. The Davis Equation (formulated by W.J. Davis Jr. of General Electric in 1926 and updated by the American Railway Engineering Association) remains the industry standard formula for calculating train resistance. It is used daily by railroad operating departments to plan train consists and fuel consumption.

Theoretical Foundations

2.1 — The Davis Train Resistance Equation

The Davis Equation calculates the total resistance force a train must overcome to maintain steady speed on level, tangent track. Resistance R (in lbs per ton) is:

$$R = 0.6 + (20/W) + 0.01V + (K \times V^2) / (W \times n)$$

Where: W = weight per axle in tons; V = speed in mph; K = air resistance coefficient (0.07 for conventional equipment, higher for double-stack intermodal); n = number of axles per car. The four terms represent: (1) journal bearing friction; (2) axle load effect; (3) flange/rail friction linear with speed; (4) aerodynamic drag quadratic with speed.

Scaling to O Scale: The model locomotive's speed must be scaled to prototype speed: V_prototype = V_model × 48 (since distance scales 1:48). A model traveling at 2 scale mph represents 96 prototype mph — faster than most freight operations. Students must carefully distinguish model measurements from prototype values throughout this module.

2.2 — Tractive Effort & the Adhesion Limit

Tractive Effort (TE) is the force a locomotive can exert at the rail. It is limited by two factors: (1) the power available from the prime mover (motor), and (2) the adhesion between wheel and rail (friction).

```
TE_motor = (HP × 375) / V   [where HP = horsepower, V = speed in mph]
TE_adhesion = μ × W_drivers   [where μ = adhesion coefficient, W_drivers = weight on driving wheels]
```

The usable tractive effort at any speed is: TE = min(TE_motor, TE_adhesion). On clean, dry rail, μ ≈ 0.30–0.35. On wet rail, μ drops to 0.18–0.25. Sand application (sanding the rail before driving wheels) raises μ temporarily.

For a model locomotive, tractive effort can be measured directly using a spring scale attached to a drawbar. Motor-limited TE is measured at low speed (stall); adhesion-limited TE is measured by loading until wheelslip occurs.

2.3 — Grade & Curve Resistance

Additional resistances must be added to the Davis base resistance for non-level, non-tangent track:

```
R_grade = 20 × G   [lbs/ton, where G = grade in percent]
R_curve = 8 / radius_degrees   [lbs/ton, where radius in degrees of curvature]
```

Total train resistance for a train of T tons on a G% grade in a D-degree curve:

```
F_total = T × (R_davis + R_grade + R_curve)   [in pounds]
```

The locomotive can move the train if and only if: TE ≥ F_total at the operating speed.

Design Scenario: A prototype 4,000 HP diesel locomotive weighing 400,000 lbs (200 tons) with 6 axles attempts to pull a 5,000-ton freight train at 30 mph up a 1.5% grade on tangent track. Using the Davis Equation (K=0.07, average car: 100 tons, 4 axles), calculate total train resistance. Calculate TE_motor at 30 mph. Calculate TE_adhesion assuming μ=0.30. Does the train make it? If not, what is the maximum tonnage this locomotive can pull at 30 mph on this grade?

2.4 — Braking Physics & Stopping Distance

Braking force on a train comes from brake shoe friction against wheel treads (conventional) or from electromagnetic/dynamic braking (modern locomotives). Total braking force:

$$F\_brake = \mu\_brake \times W\_total \quad [\text{lbs}]$$

$$\text{Deceleration: } a = F\_net / m = (F\_brake - F\_resistance) / (W/g) \quad [\text{ft/s}^2]$$

$$\text{Stopping distance: } d = V_0^2 / (2a) \quad [\text{from kinematics, level track}]$$

For a 5,000-ton train at 60 mph with μ_brake = 0.12 and total resistance 2 lbs/ton:

$$V_0 = 60 \text{ mph} = 88 \text{ ft/s}$$

$$F\_brake = 0.12 \times 10{,}000{,}000 \text{ lbs} = 1{,}200{,}000 \text{ lbs}$$

$$F\_resist = 2 \times 5{,}000 = 10{,}000 \text{ lbs}$$

$$a = (1{,}200{,}000 - 10{,}000) / (10{,}000{,}000 / 32.2) = 3.83 \text{ ft/s}^2$$

$$d = 88^2 / (2 \times 3.83) = 1{,}011 \text{ feet} \approx 0.19 \text{ miles}$$

Students compare this to published Emergency Stopping Distance tables from the FRA (Federal Railroad Administration).

2.5 — Kinetic Energy & Regenerative Braking

The kinetic energy stored in a moving train represents an enormous amount of energy — and modern locomotives increasingly recover this energy through dynamic (regenerative) braking back into the power grid or energy storage systems.

$$KE = \tfrac{1}{2}mv^2 \quad [\text{Joules}]$$

For a 5,000-ton train at 60 mph (88 ft/s = 26.8 m/s), converting mass to kg (5,000 tons × 907 kg/ton = 4,535,000 kg):

$$KE = \tfrac{1}{2} \times 4{,}535{,}000 \times 26.8^2 = 1.63 \times 10^9 \text{ Joules} \approx 453 \text{ kWh}$$

At $0.12/kWh, the recoverable energy is worth approximately $54 per stop — significant for a railroad making hundreds of stops per day per locomotive.

Laboratory Investigation

Experiment 2A — Empirical Tractive Effort Curve

Students construct an empirical TE curve for an O Scale locomotive by measuring drawbar pull at multiple speed steps using a calibrated spring scale and timing runs over a known distance.

6. Attach a spring scale to the locomotive drawbar.
7. Set throttle to speed step 5 (of 28). Allow locomotive to reach steady state. Read spring scale force while locomotive is in motion. Record force and time a 1-meter run to calculate speed.
8. Repeat for speed steps 10, 15, 20, 25, 28.
9. Plot TE (grams-force) vs. speed (scale mph, converting model speed × 48).
10. Fit a hyperbolic curve to the motor-limited region and a horizontal line to the adhesion-limited region. Identify the crossover point.
11. Convert model measurements to prototype values and compare the shape of the curve to published prototype TE curves available from locomotive manufacturer specifications.

Experiment 2B — Davis Equation Verification

Students calculate predicted train resistance using the Davis Equation for their model train (scaling parameters appropriately) and then measure actual resistance experimentally using a spring scale. Calculate percent error and propose explanations for deviation (dirty track, bearing quality, decoder back-EMF behavior).

Experiment 2C — Stopping Distance Investigation

Students accelerate the model train to a fixed speed, cut power, and measure stopping distance. Vary: (1) total train mass (add/remove cars); (2) grade (uphill vs. downhill stops); (3) track cleanliness. Plot stopping distance vs. mass and vs. grade separately. Fit curves and extract empirical friction coefficients. Compare to Davis equation predictions.

Standards & Outcomes Alignment — Module 2

Standard / Code	Disciplinary Core Idea / Practice	How the O Scale Activity Addresses It
HS-PS2-1	Analyze data supporting Newton's Second Law	Students derive acceleration from force/mass measurements and validate F=ma across multiple experimental conditions
HS-PS3-1	Create computational model of energy in fields	Students build a spreadsheet model of kinetic energy as a function of speed and mass; calculate regenerative braking energy recovery
HS-PS3-3	Design device that converts energy within constraints	Students optimize braking force for minimum stopping distance subject to wheelslip constraint (μ limit)
HS-ETS1-4	Use simulation to model proposed design solutions	Students use a Davis Equation spreadsheet to evaluate train consists before physical testing
AP Physics C Mech.	Newton's Laws, work-energy theorem, momentum, rotational dynamics	All five derivations and experiments in Module 2 map directly to AP Physics C Mechanics content
ABET (e)	Identify, formulate, and solve engineering problems	The Design Scenario in 2.3 is a complete engineering problem statement requiring formulation, calculation, and decision-making

Assessment — Module 2

Criterion	4 – Exemplary	3 – Proficient	2 – Developing	1 – Beginning
Davis Equation	Correctly calculates all four resistance terms; applies grade and curve corrections; identifies limiting	Correctly calculates three of four terms; applies grade correction; identifies one limiting factor	Correctly calculates two terms; sets up grade correction incorrectly	Cannot apply Davis Equation without step-by-step guidance

	factor (motor vs. adhesion) at each speed			
TE Curve Analysis	Plots TE curve from experimental data; fits correct curve types to motor-limited and adhesion-limited regions; converts to prototype units; compares to published spec	Plots curve; fits one region correctly; converts units	Plots curve without curve fitting; no unit conversion	Data plotted but curve is not analyzed
Braking Calculation	Correctly calculates stopping distance with all forces; derives empirical μ from experimental data; calculates KE recovery; compares to FRA tables	Correct stopping distance; derives μ; no KE calculation	Correct stopping distance formula; arithmetic errors in derivation	Cannot derive stopping distance from Newton's Laws
Error Analysis	Identifies and quantifies 3+ sources of experimental error; calculates propagated uncertainty; proposes specific improvements	Identifies 2+ sources; calculates one uncertainty	Identifies error sources qualitatively; no quantification	Errors acknowledged but not analyzed
Prototype Scaling	All calculations correctly scaled between model and prototype; units tracked throughout; scale factor applied consistently	Most calculations correctly scaled; one unit error	Scale factor applied to final answer only; intermediate steps unscaled	Prototype scaling not performed or incorrectly applied

MODULE 3

Computational Geometry & Engineering Graphics

3D Track Planning, Parametric Curves, Superelevation & CAD Clearance Analysis

Duration	120 minutes + 60-min CAD session
Level	Grade 11–12 (Pre-Calculus/Calculus, Engineering Graphics) | College Freshman (Engineering Drawing, Intro CAD)
Core Standards	HS-ETS1-2: Evaluate solutions against criteria | CCSS-Math HSG-MG (Modeling with Geometry), HSN-VM (Vector quantities)
ABET Outcomes	(b) Design experiments; (k) Use modern engineering techniques, skills, tools including CAD
Key Concepts	Coordinate geometry, parametric equations, radius of curvature, superelevation, cant deficiency, clearance envelopes, center-of-gravity analysis, 3D modeling
Required Math	Trigonometry, parametric equations, arc length integrals, coordinate transformations, vector cross products (extension)
Driving Question	How do we plan a three-dimensional railroad layout with precise clearances, smooth curves, and proper superelevation — and verify it computationally before driving a single spike?

Industry Context

Modern railroad track geometry is a rigorous engineering discipline. Track planners use specialized CAD software (Bentley OpenRail, AutoCAD Civil 3D) and track geometry cars equipped with laser sensors to measure and maintain alignment, grade, cross-level, and curvature to millimeter precision. The FRA publishes Track Safety Standards (49 CFR Part 213) specifying maximum allowable deviations for each track class. Understanding the geometry of curved track — including superelevation (cant), spiral transition curves, and clearance envelopes — is foundational to civil and transportation engineering.

Theoretical Foundations

3.1 — Coordinate Geometry of Circular Track

An O Scale curve is a circular arc. For a curve of radius R centered at (cx, cy), the track centerline follows:

$$x(\theta) = cx + R\cdot\cos(\theta)$$
$$y(\theta) = cy + R\cdot\sin(\theta)$$

The arc length L of a curve subtending angle θ (in radians):

$$L = R\cdot\theta$$

For O36 track (radius = 18 inches), a 45° section subtends π/4 radians:

$$L = 18 \times (\pi/4) = 14.14 \text{ inches} \approx 35.9 \text{ cm}$$

Converting to prototype: L_proto = 14.14 × 48 = 679 inches = 56.6 feet. Students verify this against manufacturer specifications.

3.2 — Spiral Transition Curves (Euler Spirals)

An abrupt transition from straight track to a circular curve causes an instantaneous centripetal acceleration that is uncomfortable for passengers and stresses equipment. Real railroads use a spiral transition (Euler spiral or Cornu spiral) in which curvature increases linearly from zero to the circular curve's curvature.

The Euler spiral is defined parametrically by the Fresnel integrals:

$$x(s) = \int_0^s \cos(\pi \cdot t^2/2)\, dt$$
$$y(s) = \int_0^s \sin(\pi \cdot t^2/2)\, dt$$

Where s is the distance along the spiral. Students use numerical integration (trapezoidal rule or Simpson's rule) to compute spiral coordinates for a transition from tangent to O72 curve and plot the result.

> **Calculus Extension:** Prove that the curvature $\kappa(s) = d\theta/ds$ of the Euler spiral increases linearly with s: $\kappa(s) = \pi s$. Interpret this physically: why does linearly increasing curvature minimize jerk (rate of change of acceleration)?

3.3 — Superelevation (Cant) & Cant Deficiency

On curves, the outer rail is raised above the inner rail (superelevation or cant) to counteract centripetal force and allow trains to navigate curves at higher speeds with less lateral force on wheels. The equilibrium superelevation E for a curve of radius R at speed V:

$$E = (G \times V^2) / (g \times R) \quad \text{[SI units: E in meters, G = track gauge in meters, V in m/s, R in meters]}$$
$$E_{inches} = (G_{inches} \times V^2_{mph}) / (R_{feet} \times 70.3) \quad \text{[practical US formula]}$$

Standard gauge (G = 4.708 ft = 56.5 inches). For a curve of 500-foot radius at 40 mph:

$$E = (56.5 \times 40^2) / (500 \times 70.3) = (56.5 \times 1600) / 35{,}150 = 2.57 \text{ inches}$$

In O Scale (1:48): E_model = 2.57 / 48 = 0.054 inches ≈ 1.4 mm. Students shim the outer rail of their layout curves to this calculated value.

3.4 — Clearance Envelopes

Every piece of equipment has a clearance envelope — the maximum cross-sectional space it occupies including all dynamic motions (swaying, pitching, coupler swing). Clearance diagrams are defined for each scale. In O Scale, the basic clearance envelope is 2.75 inches wide × 5.0 inches tall.

On curves, the clearance envelope shifts outward due to the chord-to-arc offset. The end overhang of a car of length L on a curve of radius R:

$$\text{Overhang} = L^2/(8R) \quad [\text{approximate for small angles}]$$

For a 15-inch passenger car on an O36 (18-inch radius) curve:

$$\text{Overhang} = 15^2/(8\times18) = 225/144 = 1.56 \text{ inches per end}$$

Total width increase = 2 × 1.56 = 3.12 inches — significantly wider than the basic envelope. This is why minimum radius requirements exist for long equipment.

Laboratory / CAD Investigation

Experiment 3A — Track Geometry Survey

Students use a digital protractor, precision ruler, and bubble level to survey an existing section of O Scale layout, recording: rail gauge at 6-inch intervals, alignment deviation from a string line, grade at 12-inch intervals using a digital level, and superelevation (cross-level) on each curve section. Data is entered into a spreadsheet and compared to published O Scale and FRA tolerance standards.

Experiment 3B — Parametric Track Planning in CAD

Using free CAD software (FreeCAD, LibreCAD, or SCARM — the last being purpose-built for model railroad track planning), students:

12. Input the room dimensions as the boundary.
13. Place track sections using parametric coordinates, calculating start/end points of each section using trigonometry.
14. Calculate the minimum passing clearance between parallel tracks on curves using the overhang formula.
15. Verify that the longest piece of equipment (specify: 15-inch passenger car) can traverse all curves without fouling adjacent track.
16. Calculate total track length, total number of track sections, and total layout area footprint.
17. Produce a dimensioned engineering drawing of the layout with all critical dimensions labeled.

Experiment 3C — Superelevation Implementation & Measurement

Students calculate required superelevation for each curve on their layout based on the intended maximum operating speed. They shim the outer rail to the calculated value using calibrated shims (business cards are approximately 0.010 inches thick — useful for shimming in sub-millimeter increments). They then operate a locomotive through the curve at the design speed and compare observed wheel flange force (estimated from sound and visual observation) to the predicted value.

Standards & Outcomes Alignment — Module 3

Standard / Code	Disciplinary Core Idea / Practice	How the O Scale Activity Addresses It
HS-ETS1-2	Evaluate competing solutions using systematic process	Students compare 3 alternative track configurations using computed clearances, length, and grade criteria
CCSS HSG-MG.1	Use geometric shapes to describe objects; apply concepts to real-world problems	Students model layout curves as circular arcs and compute arc length, overhang, and clearance analytically
CCSS HSG-MG.3	Apply geometric methods to solve design problems	Students solve the clearance problem using chord-arc geometry; solve the superelevation problem using centripetal force equations
CCSS HSF-TF	Trigonometric functions	Students compute coordinates of track endpoints using parametric trigonometry; apply arc length formulas
ABET (k)	Use modern engineering techniques, skills, and tools including CAD	Students produce a dimensioned CAD drawing of their track plan meeting drafting standards
AP Calculus AB/BC	Parametric equations, Riemann sums, Fresnel integrals (extension)	Extension 3.2 requires numerical integration of the Euler spiral using Riemann sum approximation

Assessment — Module 3

Criterion	4 – Exemplary	3 – Proficient	2 – Developing	1 – Beginning
Parametric Coordinates	All track section endpoints computed using parametric equations; coordinates verified by CAD placement; <1% dimensional error	All endpoints computed; minor arithmetic errors; verified by CAD	Major track sections computed; minor sections guessed; partial CAD verification	Track plan drawn without parametric computation; geometry not derived
Clearance Analysis	Correctly calculates overhang for all curve radii and equipment lengths; identifies all tight spots; proposes solutions	Calculates overhang for primary equipment; identifies most tight spots	Applies overhang formula to one case; cannot generalize	Cannot apply chord-arc offset formula
Superelevation	Correctly	Calculates E	Applies formula	Cannot apply

	calculates E for all curves; implements using calibrated shims; measures to within 0.5 mm; verifies at design speed	correctly; implements with minor measurement error	correctly; implementation error >1 mm	superelevation formula or does not implement
CAD Drawing Quality	Drawing meets engineering drafting standards: title block, scale notation, all critical dimensions labeled, proper line weights, North arrow	Drawing present with most elements; 2–3 drafting standards missing	Drawing present but informal; major dimensions missing	No CAD drawing produced or drawing is a sketch only
Euler Spiral (Extension)	Correctly derives parametric equations; performs numerical integration with stated step size; plots spiral; explains jerk minimization	Correctly sets up integrals; numerical result within 5% of exact	Sets up integrals; significant numerical error; no interpretation	Cannot set up Fresnel integral formulation

MODULE 4

Structural Engineering

Load Analysis, Truss Optimization & Introduction to Finite Element Concepts

Duration	120 minutes + 90-min build/test session
Level	Grade 11–12 (AP Physics, Engineering Design) | College Freshman (Statics, Mechanics of Materials)
Core Standards	HS-ETS1-1 through ETS1-4: Full engineering design cycle | HS-PS2-1: Forces and Newton's Laws
ABET Outcomes	(a)(b)(c)(e)(g)(k) — This module exercises the broadest range of ABET outcomes of the four modules
Key Concepts	Free body diagrams, method of joints, method of sections, stress and strain, factor of safety, Euler buckling, moment of inertia, deflection, FEA concepts
Required Math	Trigonometry, vector resolution, systems of equations, basic calculus (moment equations), statistics (experimental data analysis)
Driving Question	Design a bridge that spans 24 inches, supports a specified live load with a factor of safety of 3.0, minimizes material use, and survives 100 load cycles without fatigue failure. Prove your design on paper before you build it.

Industry Context

Railroad bridge engineering is governed by the American Railway Engineering and Maintenance-of-Way Association (AREMA) Manual for Railway Engineering. Chapter 15 specifies design live loads using the Cooper E series — a standardized set of hypothetical locomotives and cars that represent worst-case loading. A Cooper E-80 loading (the heaviest standard) specifies a 160,000-lb axle load. Engineers must design every bridge on every railroad to sustain Cooper E-80 loading (or lighter, by specification) with appropriate factors of safety, and must analyze fatigue life over the railroad's operational lifetime (typically 100 years).

Theoretical Foundations

4.1 — Free Body Diagrams & Support Reactions

Before analyzing any member of a structure, students must determine the support reactions at the abutments. For a simply supported beam bridge of span L with a point load P at distance a from the left support:

```
R_left = P × (L - a) / L
R_right = P × a / L
```

For a moving load (locomotive), the critical position is found by taking dR/da = 0 (maximum shear occurs when the load is at the support; maximum moment occurs when the load is at the center).

$$M_{max} = P \times L / 4 \quad \text{[at center span, for center-loaded simply supported beam]}$$

4.2 — Method of Joints for Truss Analysis

A truss is a structure of two-force members in pure compression or tension. At each joint, two equilibrium equations apply ($\Sigma F_x = 0$, $\Sigma F_y = 0$), giving 2J equations for J joints. The number of members m and reactions r must satisfy:

$$m + r = 2J \quad \text{[statically determinate condition]}$$

Students analyze a Pratt truss (vertical members in compression; diagonal members in tension) using the method of joints, solving the system of equations at each joint to find the force in every member. They then identify the most critically loaded member (highest stress) and design it accordingly.

> **Method of Joints Walkthrough:** For a 6-panel Pratt truss, span 24 inches, depth 6 inches, with center point load P = 500 grams (4.9 N): (a) calculate all support reactions; (b) solve joints from the left support inward; (c) determine which member carries the highest compressive force; (d) determine which member carries the highest tensile force; (e) if using craft sticks (cross-section 2mm × 10mm, E ≈ 4 GPa), calculate the stress in the critical member and verify it is below the yield strength (approximately 40 MPa for wood in tension).

4.3 — Euler Buckling of Compression Members

Columns and compression members can fail by buckling before reaching material yield stress. Euler's critical buckling load for a pin-ended column of length L, moment of inertia I, and modulus E:

$$P_{cr} = \pi^2 \times E \times I / L^2$$

For a craft stick in compression ($E = 4$ GPa, $I = bh^3/12 = 0.010 \times 0.002^3/12 = 6.67 \times 10^{-12}$ m^4, $L = 0.06$ m):

$$P_{cr} = \pi^2 \times 4 \times 10^9 \times 6.67 \times 10^{-12} / 0.06^2 = 73.2 \text{ N} \approx 7.5 \text{ kg}$$

Actual failure load observed in testing typically ranges from 40–80% of P_{cr} due to imperfections in real columns. Students compare theoretical and experimental buckling loads.

4.4 — Introduction to Finite Element Analysis (Conceptual)

Modern structural engineers use Finite Element Analysis (FEA) software to solve structures too complex for hand calculation. The FEA method divides the structure into small elements (finite elements), applies the stiffness matrix method to each, assembles a global stiffness matrix K, and solves the system:

$$[K]\{u\} = \{F\}$$

Where {u} is the displacement vector and {F} is the applied force vector. Students do not perform full FEA by hand, but they: (1) load a pre-built FEA model of their truss in free simulation software (e.g., SkyCiv Free Truss Calculator or MIT's web-based frame analysis tool); (2) compare FEA-predicted member forces to their hand-calculated method-of-joints results; and (3) interpret color-coded stress maps and displacement plots.

4.5 — Factor of Safety & Reliability

Engineering design is never exact — materials have variability, loads are uncertain, and failure modes are complex. The factor of safety (FoS) accounts for this uncertainty:

```
FoS = Material Strength / Applied Stress   [must be ≥ design requirement]
```

For railroad bridges, AREMA requires FoS ≥ 3.0 for steel in bending and ≥ 4.0 for tension-critical members. In this module, students must achieve FoS ≥ 3.0 in all members. This drives material quantity upward — the design challenge is minimizing total material (weight of craft sticks) while maintaining FoS ≥ 3.0 in every member. This is a classic structural optimization problem.

Laboratory Investigation

Experiment 4A — Pre-Design Analysis (Required Before Building)

Each group must complete and submit a Design Analysis Document before any construction begins. The document includes:

18. Problem statement with specified span (24 inches), design live load (instructor specifies, e.g., 1.5 kg), and required FoS (3.0 minimum).
19. Selected truss type with justification (Pratt, Howe, Warren, K-truss — research each type's advantages).
20. Dimensioned sketch with all member lengths calculated from geometry.
21. Method of joints analysis: all member forces computed and tabulated.
22. Critical member identification and stress calculation.
23. FoS verification for every member: F_applied / F_capacity ≤ 1/3 (to achieve FoS = 3).
24. Euler buckling check for all compression members longer than 2 inches.
25. Material list: number of craft sticks required, estimated bridge weight (dead load), design load-to-weight efficiency ratio.

> **Instructor Checkpoint:** Do not allow any group to begin building until the Design Analysis Document has been reviewed and approved. This enforces professional engineering practice — no construction without approved engineering drawings.

Experiment 4B — Construction & Documentation

Students build to their approved design. All deviations from the design (material substitutions, joint geometry changes) must be logged and analyzed for structural impact before construction continues. Time-lapse photos at 15-minute intervals document the build process for the final report.

Experiment 4C — Staged Load Testing

Load testing follows a staged protocol:

26. Dead load only (bridge weight): measure deflection at center span using a dial indicator or precision ruler.
27. 25% of design live load: measure deflection. Calculate theoretical deflection: $\delta = PL^3/(48EI)$ for center-loaded simply supported beam.
28. 50% of design live load: measure deflection. Check for cracking sounds or visible distress.
29. 75% of design live load: final pre-limit inspection.
30. 100% of design live load: verify bridge survives (this is the design point; bridge should not fail here if FoS ≥ 3.0 is achieved).
31. Continue loading to failure. Record failure load, failure location, and failure mode.
32. Calculate achieved FoS: F_failure / F_design_live. Compare to predicted FoS from analysis.

Load Stage	Applied Load (g)	Measured Deflection (mm)	Predicted Deflection (mm)	Observations / Distress Notes
Dead load only				
25% design load				
50% design load				
75% design load				
100% design load				
Failure load				

Experiment 4D — Fatigue Cycling (Optional Extension, 30 min)

Bridges survive not just maximum load events but thousands of load cycles. Students apply 50% of the design live load repeatedly (100 cycles of loading/unloading, using a simple lever arm mechanism) while monitoring for acoustic emissions (cracking sounds) and measuring deflection drift. This introduces the concept of fatigue life — a critical consideration in real bridge design under the AREMA specification.

Standards & Outcomes Alignment — Module 4

Standard / Code	Disciplinary Core Idea / Practice	How the O Scale Activity Addresses It
HS-ETS1-1	Define criteria and constraints with sufficient precision	The Design Analysis Document requirement enforces precise problem definition before any action is taken

HS-ETS1-2	Evaluate competing design solutions systematically	Students compare Pratt, Howe, Warren, and K-truss configurations analytically before selecting one
HS-ETS1-3	Analyze data from tests to determine similarities/differences among solutions	Inter-group comparison of failure loads, failure modes, efficiency ratios, and predicted vs. actual FoS
HS-ETS1-4	Use simulation to model effects of proposed solutions	FEA software comparison to method-of-joints hand analysis validates both approaches
HS-PS2-1	Analyze data to support Newton's Second Law	Students derive support reactions and member forces from static equilibrium (ΣF=0, ΣM=0)
ABET (c)	Design system/component/process to meet desired needs within constraints	The bridge design problem is a complete constrained engineering design problem meeting ABET (c) definition
ABET (e)	Identify, formulate, solve engineering problems	Method of joints analysis is the formulation and solution of a system of simultaneous equations representing a real engineering problem
ABET (g)	Communicate effectively	Design Analysis Document and final Engineering Report require professional technical writing and graphical communication

Assessment — Module 4

Criterion	4 – Exemplary	3 – Proficient	2 – Developing	1 – Beginning
Design Analysis Document	All 8 sections complete and correct; method of joints verified by FEA comparison; FoS ≥ 3.0 confirmed for all members; Euler buckling check performed	Sections 1–6 complete and correct; FoS confirmed for most members; buckling check for longest member only	Sections 1–4 complete; FoS calculated for critical member only; no buckling check	Document incomplete; FoS not calculated; construction began without approval
Method of Joints Accuracy	All member forces correct to within 5%; correct identification of tension/compression for all members; sign convention consistent	All member forces correct to within 10%; minor sign errors on 1–2 members	Forces correct for outer panel members; interior members have significant errors	Cannot solve 2-equation system at each joint independently
Experimental Execution	All 6 load stages executed;	5 of 6 stages completed;	3–4 stages completed;	Load testing not completed

	deflection measured to ±0.5 mm; failure load and mode precisely recorded; 100-cycle fatigue test completed	deflection measured; failure load recorded	deflection measurement inconsistent; failure load estimated	systematically
Predicted vs. Actual FoS	Predicted FoS within 20% of measured; deviation explained by specific failure mode analysis; Euler buckling vs. material failure correctly identified	Predicted within 30% of measured; general explanation offered	Prediction order-of-magnitude correct; no explanation of deviation	Prediction and measurement not compared
Engineering Report	Report in AREMA/ASCE format: problem statement, design basis, analysis, construction notes, test results, conclusions, recommendations; all figures properly captioned; references cited	Report complete; one or two format elements missing; figures present	Report present; informal format; analysis section weak; figures uncaptioned	Report is a lab notebook dump; no professional structure

MODULE CAP

Capstone Research Project

System Integration — Full Layout Engineering Package

Duration	4–6 weeks (concurrent with Modules 1–4 or following completion)
Level	Grade 11–12 | College Freshman | Suitable for Honors or Independent Study designation
Deliverables	(1) Engineering Design Package; (2) Physical Layout Demonstration; (3) Technical Presentation; (4) Individual Reflection
Team Size	3–5 students; roles assigned and rotated
Scenario	Teams receive a client brief specifying layout space, operational requirements, budget constraints, and aesthetic goals
Standards Met	All HS-ETS1 performance expectations; ABET Student Outcomes (a) through (g) and (k); CCSS-Math modeling standards HSN, HSA, HSG, HSF, HSS

The Client Brief (Sample)

Capstone Scenario — "Anthracite & Iron Railroad": Your engineering team has been contracted by the Anthracite & Iron Historical Society to design a permanently installed O Scale operating layout for the lobby of their interpretive center in Scranton, Pennsylvania. Available space: 12 feet × 8 feet. Requirements: continuous mainline operation for public viewing; a working coaling tower and water tank serviced by a steam locomotive; at least one operational freight siding; minimum two independently controlled electrical districts; one bridge spanning a simulated river gorge (minimum 18-inch span); the layout must represent northeastern Pennsylvania anthracite coal railroad operations circa 1920–1945. Budget constraint: 50 craft sticks maximum for bridge construction; all electrical wiring must meet recommended practices for permanent installations.

Engineering Design Package — Required Sections

Section 1: Project Scope & Requirements Analysis

Teams formally document all client requirements (functional, aesthetic, spatial, operational) and translate them into measurable engineering specifications. A requirements traceability matrix links each client requirement to a specific, testable specification.

Section 2: Electrical System Design (Module 1 Integration)

Complete power system design including: power district map with boundaries and justification; feeder wire routing with gauge selection and voltage drop calculations; DCC booster selection with power budget; wiring schematic to recommended practices for permanent installations; circuit breaker coordination study. A full KVL/KCL analysis of the distribution network is required.

Section 3: Operations Performance Analysis (Module 2 Integration)

Performance analysis for the primary locomotive and heaviest anticipated train consist: Davis Equation calculation for each major track section (level, maximum grade, tight curve); tractive effort curve plotted from experimental data or manufacturer specifications scaled to prototype; stopping distance calculation for the layout's longest continuous run; energy consumption estimate for a 4-hour public demonstration session.

Section 4: Track Geometry & CAD Drawing Package (Module 3 Integration)

Complete dimensioned CAD drawing set including: plan view (top) with all track centerlines, curve radii, and switch positions dimensioned; grade profile drawing showing elevation changes along the mainline; superelevation table for all curves; clearance verification for the longest equipment piece on the tightest curve; coordinate table of all major track nodes.

Section 5: Bridge Engineering Analysis (Module 4 Integration)

Complete structural analysis of the layout bridge: selected truss type with design rationale; method of joints analysis for the specified live load; FoS verification for all members; Euler buckling check for compression members; material specification; deflection prediction at design load; load test results with predicted vs. actual FoS comparison and deviation analysis.

Section 6: Integration & Risk Analysis

A synthesis section documenting how the four subsystems interact and potential failure modes at the interfaces. For example: how does the electrical system react if the bridge is over the longest feeder run (maximum voltage drop)? How does the grade on the approach to the bridge affect the required tractive effort relative to the measured TE curve? A FMEA (Failure Mode and Effects Analysis) table identifies the top 5 failure modes, their causes, effects, and mitigations.

Section 7: Construction & Test Records

A professional engineering log documenting all design deviations during construction, with analysis of structural or electrical impact. Load test results. Track geometry survey results vs. specifications. Operational test results (can the locomotive pull the specified train up the specified grade without stalling?).

Assessment — Capstone

Criterion	4 – Exemplary	3 – Proficient	2 – Developing	1 – Beginning
Requirements Traceability	All client requirements mapped to measurable specifications; each specification has a defined test method; matrix	Most requirements mapped; test methods defined for primary specs; minor gaps	Primary requirements mapped; secondary requirements not traced; test methods vague	Requirements listed but not translated to measurable specifications

	is complete and consistent			
Technical Accuracy (All 4 Modules)	All calculations in all four sections are correct; experimental results compared to theory in each section; deviations explained	Calculations correct in 3 of 4 sections; most experimental comparisons present	Calculations correct in 2 sections; experimental comparisons incomplete	Significant calculation errors across multiple sections
CAD Drawing Package	Complete drawing set to engineering drafting standards; all dimensions present; title block complete; tolerances noted where applicable	Drawing set present; most dimensions labeled; title block complete	Plan view only; major dimensions present; no profile view or superelevation table	Sketch substituted for CAD drawing; engineering drafting standards not met
Integration & FMEA	FMEA complete with 5+ failure modes; effects and causes correctly identified; mitigations are specific and implementable; cross-system interactions documented	FMEA with 3–4 failure modes; mitigations mostly specific	FMEA with 2 modes; effects and causes listed; mitigations generic	FMEA attempted but failure modes are trivial or causes are incorrect
Presentation Quality	Presentation is clear, well-organized, and professional; technical content accurate; questions answered confidently with reference to documentation; time managed effectively	Presentation clear; most technical content accurate; most questions answered adequately	Presentation organized but difficult to follow; some technical errors; some questions not answered	Presentation unorganized; technical content significantly inaccurate; questions not answered
Individual Reflection	Reflection demonstrates deep personal learning; connects module concepts to career/academic	Reflection demonstrates solid learning; connects to future goals; evaluates team performance	Reflection descriptive of activities but lacks depth; superficial connection to goals	Reflection is a summary of activities; no personal learning or evaluation evident

	goals; identifies specific moments of conceptual change; evaluates team performance honestly			

Appendix A — Reference Data

O Scale Quick Reference

Parameter	O Scale (Model)	Prototype (1:1)
Scale Ratio	1:48	—
Track Gauge	1.25 inches	4 ft 8.5 in (standard gauge)
1 model inch equals	—	4 prototype feet
1 model foot equals	—	48 prototype feet
O27 curve radius	13.5 inches	54 feet
O36 curve radius	18 inches	72 feet
O54 curve radius	27 inches	108 feet
O72 curve radius	36 inches	144 feet
DCC voltage (track)	14.5–16V AC (RMS)	—
Typical loco current draw	0.5–1.5A	—
Weight scale factor	$(1/48)^3 = 1/110{,}592$	—

Appendix B — Key Equations Summary

Module	Equation	Variables
Module 1	$V_{avg} = V_{supply} \times duty\_cycle$	V_{avg} = average motor voltage
Module 1	$V_{drop} = I \times R_{wire} = I \times (2L \times \rho/A)$	L = wire length, ρ = resistivity, A = area
Module 1	$P = IV = I^2R = V^2/R$	Standard power equations
Module 2	$R = 0.6 + 20/W + 0.01V + KV^2/(Wn)$	Davis Train Resistance (lbs/ton)
Module 2	$TE_{motor} = (HP \times 375) / V$	Tractive effort, speed in mph
Module 2	$R_{grade} = 20 \times G$	Grade resistance (lbs/ton), G in percent
Module 2	$d_{stop} = V_0^2 / (2a)$	Stopping distance, level track
Module 3	$L_{arc} = R \times \theta$	Arc length, θ in radians
Module 3	$Overhang = L^2/(8R)$	Car end overhang on curve
Module 3	$E = G \times V^2 / (R \times 70.3)$	Superelevation in inches (US units)

Module 4	$P_{cr} = \pi^2 EI / L^2$	Euler critical buckling load
Module 4	$M_{max} = PL/4$	Center moment, simply supported beam, center load
Module 4	$\delta = PL^3/(48EI)$	Center deflection, simply supported beam, center load
Module 4	$FoS = F_{strength} / F_{applied}$	Factor of safety

Appendix C — Suggested Software Tools

Tool	Module	Purpose & Access
JMRI (Java Model Railroad Interface)	1	DCC decoder programming, layout control; free at jmri.org
Analog Discovery 2 (Digilent)	1	USB oscilloscope + signal analyzer; ~$229 educational price
SCARM Track Planner	3	Purpose-built O Scale track planning CAD; free tier at scarm.com
FreeCAD	3	Full parametric 3D CAD; free and open-source at freecadweb.org
SkyCiv Truss Calculator	4	Web-based FEA truss analysis; free tier at skyciv.com
GeoGebra	2 & 3	Graphing, parametric plotting, numerical integration; free at geogebra.org
Excel / Google Sheets	All	Davis Equation spreadsheet, power budget, data tables
Python + matplotlib	2 & 3	Advanced: numerical integration of Euler spiral; data plotting

O Scale Advanced STEM Curriculum Series

Advanced STEM Curriculum Series: O Scale Model Railroading | Grades 11–12 & College Freshman

These materials may be reproduced freely for educational and non-commercial use.